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On thermodynamical properties of some coset CFT backgrounds

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Abstract: We investigate the thermodynamical features of two lorentzian signature backgrounds that arise in string theory as exact CFTs and possess more than two disconnected asymptotic regions: the 2-d charged black hole and the Nappi-Witten cosmological model. We find multiple smooth disconnected euclidean versions of the charged black hole background. They are characterized by different temperatures and electro-chemical potentials. We show that there is no straightforward analog of the Hartle-Hawking state that would express these thermodynamical features. We also obtain multiple euclidean versions of the Nappi-Witten cosmological model and study their singularity structure. It suggests to associate a non-isotropic temperature with this background.

Keywords: Conformal Field Models in String Theory, Black Holes in String Theory.

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1. Introduction

General Relativity (GR) as a low energy theory of gravity suffers from several problems. Perhaps some of these problems can be resolved in a more complete theory of gravity. String theory is an attempt to construct such a theory. At low energies string theory should however reduce in many ways back to GR and thus may well face itself those problem plaguing low energy gravity. It seems that if string theory does play a role in fixing problems in gravity it will do it by adding a "twist" to GR — a new point of view that could have been in principle envisaged already in a GR framework.

The addition of massless particles motivated by string theory does help in resolving some of the central problems in GR, such as those related to the presence of singularities. Another manner in which string theory can influence GR is by enlarging the portion of space-time arena which one would need to probe. For example, the lorentzian uncharged black-hole string background contains two disconnected static regions, which we shall somewhat loosely call boundaries. There is also a region between the horizons and the singularities. A charged black hole (CBH) in GR has an inner horizon, an outer horizon and a time-like singularity [1]. A CBH obtained from string theory has in addition asymptotic regions that extend beyond its time-like singularities [2]. A similar enlargement occurs in the case of exact string backgrounds that describe a closed cosmology. There one is led in string theory to a picture in which the closed cosmology is embedded within static non-compact regions called "whiskers" [3].

What is common to all these lorentzian examples is that they have two or more asymptotic regions [3, 2, 4]. In string theory it is natural to study these extended space-time regions. It turns out that one can calculate the scattering matrix for waves to pass from the whiskers through the big bang/big crunch space-like singularities [3, 4] as well as beyond the time-like CBH singularities [2, 5]. All these calculations do not take into account the possible back reaction which in other cases have shown to be fatal [6, 7] (for a more complete list of references see [8]). In fact, one usually expects that back reaction will eventually detach the regions of space-time beyond a big crunch singularity as well as those regions beyond a CBH's singularity. Actually, one would expect already the regions beyond its inner horizon to decouple [9, 10]; see however [11].

String theory could assist in evaluating the actual role of back reaction if/once a holographic description is found. In its absence, another way to assess the importance of back reaction could be addressed by considering the appropriate euclidean version of the system [12]. Once the propagation on such a background is found to be free of singularities, one may investigate the manner in which the more elaborate lorentzian structure is encoded in the euclidean one. One may also attempt to determine the extent to which it can be used to detect effects of singularities.

In the case of an uncharged AdS black hole it has been shown that the effect of having two lorentzian boundaries is equivalent to having a system with an entangled pure state [13, 14]. This is encoded in the euclidean version through the emergence of thermodynamical properties such as the temperature of the black hole. Such properties are not associated with a usual vacuum state.

In this note we wish to investigate the thermodynamical properties of euclidean versions of lorentzian systems that have more than two asymptotic regions. It could be that in the presence of horizons such cases can be described by pure entangled states. It may also turn out that such systems posses more than one euclidean version.

In section 2 we discuss the euclidean versions of two-dimensional CBHs corresponding to exact quotient CFTs. These systems have various boundaries separated by both an outer (event) and an inner (Cauchy) horizon. We find them to have two euclidean versions, each with its own associated temperature [15], specific heat and electro-chemical potential. The electro-chemical potential is related to the value of a Wilson loop. We further discuss a lorentzian derivation of the temperatures associated with the two different euclidean versions.

In section 3 we investigate the quantization of a scalar field in the extended 2-d CBH background. We describe an attempt to find a simple generalization of the Hartle-Hawking state for the extended CBH — a system which a priori has at least four asymptotic regions. We show, under certain assumptions, that a class of such states does not exist.

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In section 4 we discuss the euclidean version of a cosmological example in which there are several lorentzian boundaries separated by big bang/big crunch singularities and not by horizons [16, 3]. This is also an exact coset CFT which describes a cosmology with whiskers. We obtain its euclidean version by analytic continuation, and discuss its singularity structure as well as possible non-isotropic thermodynamical properties. The same answer is also obtained by doing a euclidean coset.

In appendix A we extend the discussion of the relation between the value of a Wilson loop and the value of the chemical potential conjugate to angular momentum in Kerr black holes. In appendices B, C we elaborate on the tools used in the discussion of quantization of a scalar field in the CBH background.

2. Charged 2-d black holes

2.1 Lorentzian charged 2-d black hole — a review

A class of charged 2-d black hole backgrounds [17] arising from coset CFT's [18] was recently studied in [2]. One begins with a three-dimensional background arising by gauging [19] a WZW model $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$ by a non-compact U(1) subgroup. The gauge group action can be chosen as [2]

$$\Big(g, x_L, x_R\Big) \mapsto \Big(e^{\rho\sigma_3/\sqrt{k}} g e^{\tau\sigma_3/\sqrt{k}}, x_L + \rho', x_R + \tau'\Big)\,. \tag{2.1}$$

Here $g \in \mathrm{SL}(2,\mathbb{R})$, $x \in \mathrm{U}(1)$ and $k > 0$ is the $\mathrm{SL}(2,\mathbb{R})$ level. Picking a single non-anomalous U(1) gauge group can then be achieved by imposing the constraints

$$\underline{\rho} = R\underline{\tau}\,, \tag{2.2}$$

where $\underline{\tau} = (\tau, \tau')$, $\underline{\rho} = (\rho, \rho')$, and R is an SO(2) matrix

$$R = \begin{pmatrix} \cos(\psi) & \sin(\psi) \\ -\sin(\psi) & \cos(\psi) \end{pmatrix}. \tag{2.3}$$

In addition to (2.2) we set

$$\underline{\tau} = |\tau|(1, 0)\,. \tag{2.4}$$

Gauging with respect to the so defined U(1) defines a family of coset CFT's $\frac{\mathrm{SL}(2,\mathbb{R})\times \mathrm{U}(1)}{\mathrm{U}(1)}$ labelled by the angular parameter ψ. The metric and dilaton backgrounds can be obtained from the gauged WZW model action by fixing the gauge and then integrating the gauge fields out in the path integral. One can further perform a Kaluza-Klein (KK) reduction along the x-direction using the facts that the x direction can be chosen relatively very small and that in the large k limit, which is of interest to us, low energy gravity is a good approximation. Upon the KK reduction one obtains a background describing a two-dimensional CBH. The corresponding Penrose diagram contains six basic regions that are periodically repeated in the maximally extended solution.

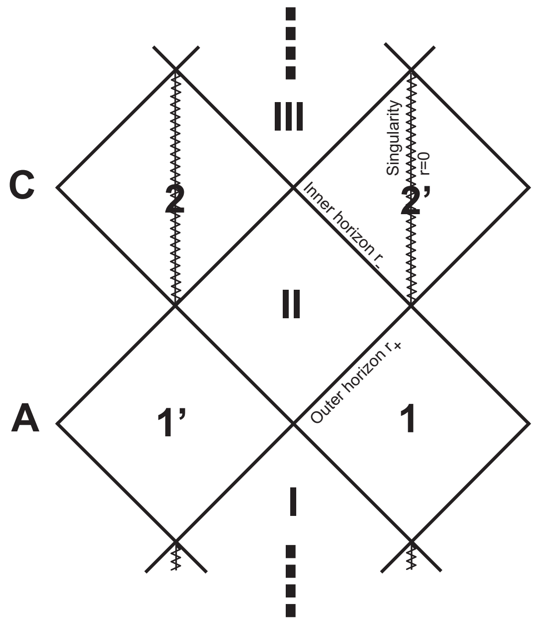

Figure 1: Penrose diagram of the 2-d charged black hole.

We will concentrate on the two static regions that are analogs of the regions outside the outer and and beyond the inner horizons in the four-dimensional Reissner-Nordstrom (RN) solution. A smooth region, to which we will refer to as a region of type A, is represented on the Penrose diagram (figure 1) by the union of regions 1 and 1' while a region containing the singularity, to be referred to as type C region, is represented by the union of regions 2 and 2'. We parameterize the corresponding regions in $\mathrm{SL}(2,\mathbb{R})$ by

$$g_A = e^{\frac{1}{2}(z+y)\sigma_3} e^{\theta\sigma_1} e^{\frac{1}{2}(z-y)\sigma_3}\,, \qquad g_C = e^{\frac{1}{2}(z+y)\sigma_3} e^{\theta\sigma_1} i\sigma_2 e^{\frac{1}{2}(z-y)\sigma_3}\,, \tag{2.5}$$

where the coordinates θ, z and y here range from $-\infty$ to $+\infty$, and fix the gauge by setting $z = 0$ (which fixes the gauge completely since z is a non-compact direction in $\mathrm{SL}(2,\mathbb{R})$). The background values of the metric in these two regions were found in [2] to be

$$\text{Region of type A}: \ \frac{1}{k}ds^2 = d\theta^2 - \frac{\coth^2(\theta)}{(\coth^2(\theta) - p^2)^2}dy^2 \tag{2.6}$$

$$\text{Region of type C}: \ \frac{1}{k}ds^2 = d\theta^2 - \frac{\tanh^2(\theta)}{(\tanh^2(\theta) - p^2)^2}dy^2 \tag{2.7}$$

where

$$p^2 = \tan^2\left(\frac{\psi}{2}\right). \tag{2.8}$$

For an arbitrary value of p (ψ) one of the two regions contains a time-like singularity line on which the g_{yy} component of the metric blows up. Assuming for definiteness $0 \leq \psi \leq \pi/2$, that is $p^2 \leq 1$, we have a singularity in region C and a smooth region A. The dilaton and

U(1) gauge field backgrounds read[1]

$$\begin{aligned}\text{Region of type A}: \quad \Phi &= \Phi_0 - \frac{1}{4}\log\left(\frac{1}{1-p^2} + \sinh^2(\theta)\right)^2 \\ A_y &= \frac{\sqrt{k}p}{p^2 - \coth^2(\theta)}\end{aligned} \tag{2.9}$$

$$\begin{aligned}\text{Region of type C}: \quad \Phi &= \Phi_0 - \frac{1}{4}\log\left(\frac{1}{1-p^2} - \cosh^2(\theta)\right)^2 \\ A_y &= \frac{\sqrt{k}p}{1 - p^2\coth^2(\theta)}.\end{aligned} \tag{2.10}$$

Here Φ_0 is a constant and the dilaton background value is normalized so that $g_s = e^{\Phi}$.

The usual metric of the 2-d CBH (in Schwarzschild-like coordinates) is obtained by the following coordinate transformation (in regions of type A):

$$t = \frac{2y}{1-p^2}, \qquad r = \frac{2e^{-2\Phi}}{\sqrt{k}}. \tag{2.11}$$

In terms of r and t the metric and gauge field are:

$$\frac{4}{k}ds^2 = -f(r)dt^2 + \frac{dr^2}{r^2 f(r)}, \qquad A_t = \frac{Q}{2r}, \tag{2.12}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \tag{2.13}$$

The parameters M and Q are related to p and Φ_0 as follows

$$M = \frac{e^{-2\Phi_0}}{\sqrt{k}}\frac{1+p^2}{1-p^2}, \qquad Q = \frac{2e^{-2\Phi_0}}{\sqrt{k}}\frac{p}{1-p^2}. \tag{2.14}$$

The singularity is located at $r = 0$. The horizons are located at

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2}. \tag{2.15}$$

Note that

$$p^2 = \frac{r_-}{r_+}. \tag{2.16}$$

The ADM mass and charge observed from the asymptotic infinity of regions 1 and 1' are M and Q, while the ones observed from regions 2 and 2' are $-M$ and Q.

2.2 Euclidean charged 2-d black hole background

The euclidean 2-d charged black hole (CBH) is given by gauging a time-like $\widetilde{U}(1)$ in $\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times$ U(1), where ($\sim$) stands for the universal cover group. The group action is

$$(g, x_L, x_R) \mapsto \left(e^{i\rho\sigma_2/\sqrt{k}} g e^{i\tau\eta\sigma_2/\sqrt{k}}, x_L + \eta\rho', x_R + \tau'\right), \tag{2.17}$$

[1] Here and below the gauge field is normalized so that its contribution to the effective action is $\frac{1}{4}\int dx^2\sqrt{-g}e^{-2\Phi}F^2$.

where $\underline{\tau} = |\tau|(1,0)$, $\eta = 1$ corresponds to axial type gauging and $\eta = -1$ to the vectorial one. The anomaly cancellation condition now implies

$$\underline{\rho} = R\underline{\tau} \tag{2.18}$$

with

$$R = \begin{pmatrix} \cosh(\psi_E) & \sinh(\psi_E) \\ \sinh(\psi_E) & \cosh(\psi_E) \end{pmatrix}. \tag{2.19}$$

In global coordinates, $g \in \mathrm{SL}(2,\mathbb{R})$ is represented as follows

$$g = e^{\frac{i}{2}(\chi+\phi)\sigma_2} e^{\theta\sigma_1} e^{\frac{i}{2}(\chi-\phi)\sigma_2}. \tag{2.20}$$

To get a smooth quotient geometry we need to gauge the universal cover of $\mathrm{SL}(2,\mathbb{R})$ ($\widetilde{\mathrm{SL}}(2,\mathbb{R})$), which is obtained by unwrapping the coordinate χ. We fix the gauge by setting $\chi = 0$.[2] As in the lorentzian case, we do a KK reduction of the three dimensional coset along the compact x direction. We obtain the following two dimensional metric, dilaton and gauge field:

$$\frac{1}{k}ds^2 = d\theta^2 + \frac{\coth^2(\theta)}{(\coth^2(\theta) + p_E^{2\eta})^2} d\phi^2\,, \tag{2.21}$$

$$\Phi = \tilde{\Phi}_0 - \frac{1}{2}\log(p_E^{2\eta}\cosh^2(\theta) + \sinh^2(\theta))\,, \tag{2.22}$$

$$A_\phi = \frac{\sqrt{k}p_E^\eta}{\coth^2(\theta) + p_E^{2\eta}}\,. \tag{2.23}$$

The euclidean time variable ϕ has a canonical periodicity 2π.[3] The parameter p_E^2 equals

$$p_E^2 = \tanh^2\left(\frac{\psi_E}{2}\right) \tag{2.24}$$

This geometry looks like an infinite cigar which, for $\eta = -1$, develops a bump at $\coth^2(\theta_b) = 1/p_E^2$ (see figure 2). This euclidean geometry (2.21) is obtained from the lorentzian one (2.6) by the following Wick rotations:

$$\begin{aligned} &\text{Region A } (\eta = 1): && y \to i\phi\,, && \psi \to i\psi_E && (p^2 \to -p_E^2) \\ &\text{Region C } (\eta = -1): && y \to ip_E^2\phi\,, && \psi \to i\psi_E && (p^2 \to -p_E^2) \end{aligned} \tag{2.25}$$

The euclidean metric of region A then arises from the axial gauging ($\eta = 1$) while the one of region C from the vectorial gauging ($\eta = -1$). The corresponding euclidean CFT's are related to each other by a T-duality that acts as

$$T : \psi_E \mapsto i\pi - \psi_E$$

[2] If instead we fix the gauge by setting $\phi = 0$, we remain with a residual discrete identifications as in [3].

[3] If we gauge a finite cover of $\mathrm{SL}(2,\mathbb{R})$, instead, we obtain an orbifold of (2.21) with a conical singularity at $\theta = 0$.

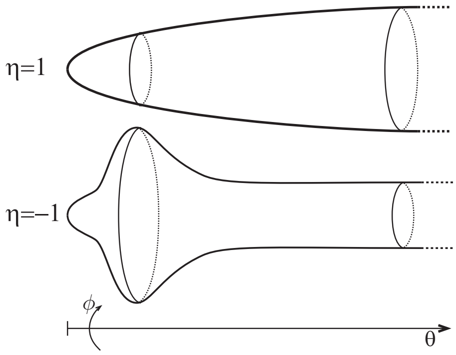

Figure 2: Euclidean charged black hole.

and exchanges an axial type gauging with a vectorial one.[4] Under this transformation[5]

$$T : p_E^2 \mapsto \frac{1}{p_E^2} \qquad (r_+ \leftrightarrow r_-) \,. \tag{2.26}$$

From the above euclidean metrics we read off the asymptotic radii of the euclidean time direction ϕ:

$$R_A^2(\theta \to \infty) = \frac{k}{(1+p_E^2)^2} \,, \qquad R_C^2(\theta \to \infty) = \frac{k p_E^4}{(1+p_E^2)^2} \,. \tag{2.27}$$

This suggests that the temperatures

$$\begin{aligned} T_A &= \frac{1}{2\pi\sqrt{\alpha' k}}(1-p^2) = \frac{1}{2\pi\sqrt{\alpha' k}}\frac{r_+ - r_-}{r_+} \\ T_C &= \frac{1}{2\pi\sqrt{\alpha' k}}\frac{(1-p^2)}{p^2} = \frac{1}{2\pi\sqrt{\alpha' k}}\frac{r_+ - r_-}{r_-} \end{aligned} \tag{2.28}$$

should be associated with the respective static regions in the Minkowski space-time. Here we have restored the dimensionfull unit – the string length scale $\sqrt{\alpha'}$ — using the fact that $R_{AdS_3}/\sqrt{\alpha'} = \sqrt{k}$.

When we take the uncharged limit $p,\ p_E \to 0$ (equivalently, when the inner horizon coincides with the singularity $r_- \to 0$), the euclidean metric outside the black hole event horizon becomes (see eq. (2.21) with $\eta = 1$):

$$\frac{1}{k} ds^2 = d\theta^2 + \tanh^2(\theta) d\phi^2 \,. \tag{2.29}$$

The temperature of the uncharged black hole, as read from (2.29), (2.28), is $T_A = (2\pi\sqrt{\alpha' k})^{-1}$. On the other hand, the $p,\ p_E \to 0$ limit of the region beyond the singularity is degenerate (see eq. (2.21) with $\eta = -1$):

$$\frac{1}{k} ds^2 \to d\theta^2 + p_E^4 \coth^2(\theta) d\phi^2 \,, \qquad p_E \to 0 \,. \tag{2.30}$$

[4]The vector-axial symmetry can be manifested for example by an element of $O(2,2,\mathbb{Z})$ (for a review, see [20]) together with a certain orbifolding in the case $\sqrt{k} p_E \in \mathbb{Z}$. The orbifolding is expected to be a symmetry of the CFT as for compact parafermions.

[5]Note that in the analytic continuation to Minkowski space the T-duality transformation involves in addition to $\psi \mapsto \pi - \psi$ a rescaling of the y coordinate by p^2 which is a valid operation because y is noncompact in that case. This rescaling results from changing the gauge fixing in (2.5) from $z = 0$ to $y = 0$.

This background is ill defined for two reasons. First, formally, the radius of the euclidean time direction ϕ vanishes as p_E approaches zero (which implies, in particular, $T_C \to \infty$). Second, there is a curvature singularity at $\theta = 0$. Hence, the temperature T_C associated with the region beyond the singularity is ill defined in the uncharged limit — there is no horizon and no straightforward way to define a smooth euclidean continuation of the region beyond the singularity of the uncharged black hole.[6] This result is perhaps compatible with the expectation [22, 2, 5] that in the uncharged case the singularity (as seen from the asymptotically flat region behind it) is (classically) a perfect reflector and hence the low energy physics beyond the singularity cannot communicate with any other region; in particular, this suggests that there is no entanglement between regions 2 and 2'.

The emergence of a compact dimension in the presence of a gauge potential suggests that a thermodynamical characteristic could be attributed to the value of a Wilson loop around the compact direction (see [25] for different view on the same point). In search for such a meaning one notes that another thermodynamical characteristic of the CBH is its electric chemical potential. Adding a charge q point-like particle at rest in the black hole frame at θ_0 corresponds to adding a charge density[7]

$$J^0 = q\frac{\delta(\theta-\theta_0)}{\sqrt{-g}}\,, \qquad J^i = 0\,,$$

which contributes to the euclidean action

$$-\int d^2x\sqrt{-g}J^\mu A_\mu = -q\oint d\phi A_\phi(\theta_0)\,. \tag{2.31}$$

If (following [26, 27, 28]) we identify the value of the action on the equations of motion with the thermodynamical free energy

$$-I_{|\text{on shell}} = \beta F = \beta M - S - \beta\sum_i \mu_i \mathcal{Q}_i\,,$$

then the Wilson loop (2.31) at infinity is given by

$$W = \exp\left(\oint d\phi A_\phi(\theta\to\infty)\right) = \exp(\beta\mu_{\text{el}})\,. \tag{2.32}$$

In the lorentzian geometry the value of the gauge field at infinity could have assumed ab initio any value; different values are related by large gauge transformations. In the

[6]Note that when $p_E = 0$ and $\eta = -1$, χ cannot be changed by a gauge transformation and hence $\chi = 0$ cannot be fixed in (2.20). Instead, one can fix the gauge by setting $\phi = 0$. The metric one gets is: $\frac{1}{k}ds^2 = d\theta^2 + \coth^2(\theta)d\chi^2$. If, for instance, we gauge $\mathrm{SL}(2,\mathbb{R})$ instead of $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ then χ is compact with periodicity 2π, and this background looks like a trumpet [21, 22] which degenerates at $\theta = 0$ and with asymptotic radius $R_\infty/\sqrt{\alpha'} = \sqrt{k}$. But in any case, since there is no tip and hence no conical defect, it is not well defined a priori which is the appropriate euclidean background obtained by the Wick rotation of the region beyond the singularity of the uncharged black hole [23, 24, 21, 22, 20].

[7]In general for a point-like particle $J^\mu(\mathbf{x}) = q\int \frac{\partial y^\mu}{\partial\tau}\frac{\delta^d(\mathbf{x}-\mathbf{y}(\tau))}{\sqrt{-g}}d\tau$, where τ is its proper time. The corresponding contribution to the action is $\delta I = -\int dx^d\sqrt{-g}J^\mu A_\mu = -q\int\frac{\partial y^\mu}{\partial\tau}A_\mu d\tau$, which for a particle at rest equals $-q\oint dy^0 A_0(\vec{r}_0)$.

euclidean geometry the angular part of the vector field A_μ (which is A_ϕ) has to vanish at the tip of the cigar $\theta = 0$. This euclidean regularity condition at the tip dictates a particular non-vanishing constant value of A_ϕ at infinity. The value obtained is the same one obtained from the euclidean coset,

$$A_\phi^{(A)}(\theta \to \infty) = A_\phi^{(C)}(\theta \to \infty) = \frac{\sqrt{k} p_E}{p_E^2 + 1}\,, \tag{2.33}$$

which leads to the following values of the Wilson loops:

$$W_A = W_C = \exp\left(\frac{2\pi \sqrt{k} p_E}{p_E^2 + 1}\right)\,. \tag{2.34}$$

Combining (2.28) and (2.34) one reads the corresponding chemical potentials:

$$\mu_A = -p = -\frac{Q}{r_+}\,, \qquad \mu_C = \frac{1}{p} = \frac{Q}{r_-}\,, \tag{2.35}$$

which are in agreement with [28]. In the lorentzian space this electro-chemical potential is the electric potential on the horizon of the black hole (outer and inner, respectively).

One can repeat the computation for a RN black hole in an arbitrary dimension $d \geq 4$. When analytically continuing a region outside of the black hole to a compact euclidean geometry the horizon becomes the tip of the d-dimensional cigar.[8] Naively, the value of A_0 at the tip is the electric potential at the horizon, but since A_μ is a vector field its angular component has to vanish at the tip. We learn that in order to have a non-singular gauge field at the euclidean tip, in the Lorenzian black hole we have to do a large gauge transformation which gives a vanishing gauge field at the horizon (which can be done in the Lorenzian case). After the large gauge transformation the gauge field will not vanish at infinity anymore (for $d \geq 4$), but instead its value at infinity is minus the electric potential at the horizon. After analytic continuation to the euclidean cigar this asymptotic value of the gauge field at infinity leads to a non-trivial Wilson loop. The value of the Wilson loop is minus the electric potential at the horizon times the circumference of the compact time, which is the inverse black hole temperature. This computation gives the following electro-chemical potential for an RN black hole in dimension $d \geq 4$:

$$\mu_{\rm el} = -\frac{Q}{r_+^{d-3}}\,. \tag{2.36}$$

Requiring a non-singular metric for a euclidean black hole led to a compact euclidean dimension. Requiring a non-singular gauge field (at the tip) led to a non-trivial Wilson loop at infinity. This compact dimension and a non-trivial Wilson loop both reflect the surprising feature of black holes — they behave as a thermodynamical system. The temperature associated with them is encoded in the asymptotic radius of the compact direction and the electro-chemical potential is encoded in the asymptotic Wilson loop wrapped around the compact direction. In a similar way one can compute the chemical potential for rotating particles in a Kerr black hole; this is demonstrated in appendix A.

[8]The euclidean geometry is a cigar-like in the radial and euclidean time coordinates times a $(d-2)$-dimensional sphere which shrinks to zero size at the tip.

Back to 2-d, another thermodynamics characteristic of the 2-d CBH is its entropy. By integrating the equations

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_Q , \qquad \mu = -T \left(\frac{\partial S}{\partial Q}\right)_M , \tag{2.37}$$

where $E_A = M^{(A)}_{\rm ADM} = M$ and $E_C = M^{(C)}_{\rm ADM} = -M$, we find

$$S_A = \pi\sqrt{k}\, r_+ , \qquad S_C = \pi\sqrt{k}\, r_- . \tag{2.38}$$

The heat capacities in regions A and C are read from the relation $C = \left(\frac{\partial E}{\partial T}\right)_Q = T\left(\frac{\partial S}{\partial T}\right)_Q$:

$$C_A = \frac{\pi\sqrt{k}(r_+ - r_-)}{2}\frac{r_+}{r_-} , \qquad C_C = -\frac{\pi\sqrt{k}(r_+ - r_-)}{2}\frac{r_-}{r_+} . \tag{2.39}$$

Note that $S_C \neq S_A$ (except for the extremal case $M = |Q|$ $(r_+ = r_-)$), and $C_C \neq C_A$. Moreover, C_A is positive while C_C is negative. The quantities S_C and C_C are, however, in agreement with T-duality (2.26), which interchanges $r_+ \leftrightarrow r_-$.[9]

To summarize, low energy observers beyond the singularity see a negative mass object with a naked singularity, with a negative specific heat and an entropy which is lower than the black hole entropy as seen by an observer outside the event horizon. The interpretation of these results is not clear to us, though it is possible that string theory manages to "take care of itself" even in such a situation; the agreement between the thermodynamical and the T-duality derivations motivates this logical possibility.[10]

Note that the T-duality we have been exploiting when comparing the thermodynamical quantities is a duality between euclidean backgrounds. Thus, a priori it does not have to be a true symmetry of the Minkowski signature background.[11] However, even if we assume it is a symmetry (not relying on a would be thermodynamics of negative mass naked singularities), it is not obvious that the total entropy of the two euclidean systems measures the same thing. If nevertheless the entropy turns out to be T-duality invariant it should be expressed in terms of a symmetric function in S_A and S_C, such as for instance their sum. This would imply that there is more entropy to the system than seen above in a single region. A different possibility, argued in [30], is that the total entropy of the system is the (absolute value of the) difference (this result assumed that degrees of freedom do not reside beyond the inner horizon).

[9] Or, equivalently, $M \to -M$ (recall that $r_\pm = |M \pm \sqrt{M^2 - Q^2}|$).

[10] T-duality implies that while the correlators of momentum modes on the euclidean background corresponding to region A ((2.21) with $\eta = 1$) describe, via analytic continuation, the physics of momentum modes outside the event horizon of the lorentzian background, the correlators of winding modes in A describe instead, via analytic continuation, the physics of momentum modes in the region beyond the inner horizon. Alternatively, the latter is described by analytic continuation of momentum modes on the T-dual euclidean background corresponding to region C ((2.21) with $\eta = -1$). The mysterious thermodynamical quantities, in the region with $\eta = -1$, are obtained from this T-dual euclidean description.

[11] One could use T-duality along the time-like direction of the Minkowski signature background. This T-duality however is known to suffer, in some cases, from various kinds of problems, see e.g. [29].

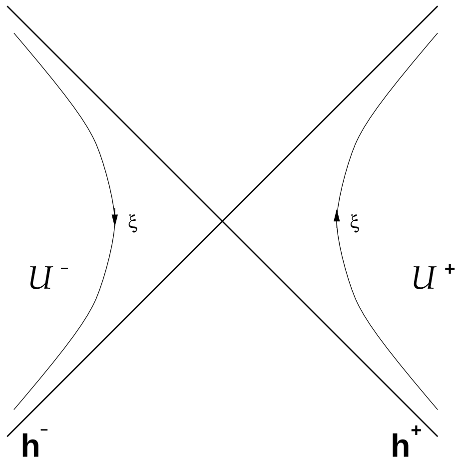

Figure 3: Bifurcate Killing horizon.

2.3 Minkowski signature computation of the temperatures

For the model at hand we know explicitly the scalar field (tachyon) wave functions in all space-time regions including the type C static region with singularity.[12] The wave functions for 2-d CBH as well as a scattering problem set in the region behind the singularity were recently discussed in [2]. The wave functions essentially descend from the Laplace eigenfunctions on the $\mathrm{SL}(2,\mathbb{R})$ group manifold. In particular, they are smooth on the singularity lines [2, 5]. One can therefore perform an alternative computation of the temperatures in regions A and C via Bogolubov transformations relating the analogs of Rindler and Minkowski vacua on outer and inner horizons, respectively. A computation of this kind was done by Israel in [13] for eternal 4-d black holes. The presence of charge does not alter the computation — the essential feature being the fact that the Killing vector is globally time-like in a static region on each side of the horizon (see e.g. [31]). If one considers charged particles on the background at hand the corresponding electro-chemical potential (2.35) will enter the canonical distribution. Here we are interested mainly in the uncharged field case characterized by an Hawking temperature only. The arguments of [13] are then directly applicable to region A or region C separately.

Let us outline here the ingredients of the computation in [13]. We consider a static region of space-time that consists of two patches: $\mathcal{U}^-, \mathcal{U}^+$ — the left and the right wedges separated by a bifurcate Killing horizon. In two dimensions the last one is comprised out of two intersecting light rays $\mathbf{h}^+$, $\mathbf{h}^-$ (see figure 3). The Killing vector ξ in $\mathcal{U}^-$ and $\mathcal{U}^+$ is assumed to be everywhere time-like. One considers then two quantizations in $\mathcal{U}^- \cup \mathcal{U}^+$. One is built upon "Killing modes" which are defined as energy eigenfunctions of the Klein-Gordon equation as registered by a Killing orbit observer in each patch $\mathcal{U}^\pm$. The corresponding one-particle Hilbert spaces $\mathcal{H}^-$, $\mathcal{H}^+$ consist of solutions which are purely negative frequency with respect to Killing time translations.[13] Thus the second quantized

[12]At the moment, this is not the case with the 4-d Reissner-Nordstrom solution.

[13]The positive/negative frequencies in region $\mathcal{U}^-$ are defined w.r.t. $-\xi$.

Fock space defined by Killing time modes can be written as a direct product

$$\mathcal{F}(\mathcal{H}^-) \otimes \mathcal{F}(\mathcal{H}^+) .$$

The corresponding vacuum $|0\rangle_- \otimes |0\rangle_+$ is an analog of the Boulware vacuum of an extended Schwarzschild solution. In the other quantization an analog of the Hartle-Hawking vacuum $|HH\rangle$ is defined via Klein-Gordon eigenfunctions whose restrictions on the light rays $\mathbf{h}^+$, $\mathbf{h}^-$, comprising the horizon, contain positive frequency modes with respect to the affine parameters along the rays. It follows from a standard computation [13] of Bogolubov coefficients relating the two quantizations that the Hartle-Hawking vacuum can be represented as an entangled state

$$|HH\rangle = \prod_{\omega,q} \sum_{n=0}^{\infty} \exp\left(-\frac{n\pi}{\kappa}(\omega - \mu_{\rm el} q)\right) |n,\omega,q\rangle_- \otimes |n,\omega,q\rangle_+ \in \mathcal{F}(\mathcal{H}^-) \otimes \mathcal{F}(\mathcal{H}^+) , \quad (2.40)$$

where $|n,\omega,q\rangle_\pm$ stands for a state with n "Killing particles" with energy ω and charge q. Here κ is the surface gravity constant characterizing the Killing horizon at hand and $\mu_{\rm el}$ is the electric potential (at the horizon). Restriction of the state (2.40) to a single patch $\mathcal{U}^+$ (or $\mathcal{U}^-$) yields a thermal density matrix characterized by an Hawking temperature

$$T = \frac{\kappa}{2\pi} . \quad (2.41)$$

The main assumption in the computation leading to (2.41) is that the horizon is a non-singular part of space-time. The analog of Hartle-Hawking vacuum (2.40) can be characterized by the requirement of its invariance under the isometries generated by the Killing vector at hand and by the regularity of the scalar field stress-energy tensor on the horizon (see e.g. [31] for a detailed discussion).

The above considerations, being applied separately to region A or C of the 2-d CBH solution, yield Hawking temperatures $T_{A,C} = \frac{\kappa_{A,C}}{2\pi}$, where $\kappa_{A,C}$ is the surface gravity of the outer (for region A) or inner (for region C) horizon. Noting that the Killing vector, normalized to have norm one at infinity, is in both regions

$$\xi \equiv \pm \frac{(1-p^2)}{\sqrt{k}} \frac{\partial}{\partial y} , \quad (2.42)$$

one can read off the surface gravity from the asymptotic expansion of the metric in the vicinity of the corresponding horizons:

$$\begin{aligned} \text{Region of type A}: \ & \frac{1}{k} ds^2 \approx d\theta^2 - \theta^2 dy^2 \\ \text{Region of type C}: \ & \frac{1}{k} ds^2 \approx d\theta^2 - \frac{\theta^2 dy^2}{p^4} . \end{aligned} \quad (2.43)$$

One obtains then the same result as in (2.28), as expected.

The temperatures considered above correspond to the Hawking radiation temperatures as measured by asymptotic observers in region A and C. In addition to that, one can consider temperatures measured locally by observers moving along Killing vector orbits.

Those temperatures are obtained by dividing the asymptotic temperatures by the Killing vector norm. We then have

$$T_A^{\rm local} = \frac{|\coth^2(\theta) - p^2|}{2\pi\sqrt{\alpha' k}\coth(\theta)}, \tag{2.44}$$

$$T_C^{\rm local} = \frac{|\tanh^2(\theta) - p^2|}{2\pi\sqrt{\alpha' k}p^2\tanh(\theta)}. \tag{2.45}$$

We see that both temperatures characteristically go to infinity on the corresponding horizons. In addition, in region C the local temperature vanishes on the singularity line.

In this section we have followed the point of view of [13] in which the boundary conditions were set up on the horizon. Note that a situation with a clear separation between the Hilbert space two components is considered in [14]. A pair of Hilbert spaces of regions 1 and 1' will be entangled with the Boltzman weight appropriate for the temperature T_A (2.28). Since regions 1 and 1' are outside the event horizons, while regions 2 and 2' lay beyond the inner horizons, a common wisdom would be that the latter regions are not involved in the former entanglement. An alternative entangled state could be built out of states belonging to the Hilbert spaces of regions 2 and 2' and weighted in that case by the Boltzman factor corresponding to the temperature T_C (2.28). Such states may well reproduce correlation functions between operators evaluated either only in regions 1 and 1' or only in regions 2 and 2'. On the other hand, to reproduce mixed correlators such as one between an operator inserted in region 1 and another inserted in region 2, one has to quantize the field in all regions. An attempt to do this is the subject of the next section.

3. Quantization of a scalar field in the extended charged BH background

The discussion in the previous section concerned with the static patches $1 \cup 1'$ and $2 \cup 2'$, each one considered separately from all other regions in the extended Penrose diagram of figure 1. In this section we consider a scalar field quantization in the space-time region that includes all together regions $1, 1', 2, 2'$ and II of figure 1. These regions, together with region III, represent a basic block of the maximally extended space-time (that includes infinitely many blocks of the same kind). The space-time region at hand is not globally hyperbolic and is not everywhere static.[14] We thus have to deal with a rather non-standard situation here. We will present two possible approaches to quantization on this space-time. The first one follows more closely the common wisdom of canonical quantization in curved space-time, while the second one relies on the $\mathrm{SL}(2,\mathbb{R})$ structure underlying the model at hand and is more string theory inspired.

3.1 Quantization 1

Let us start by outlining the main steps in which we will proceed in this subsection. First we discuss general features of the quantization on the extended space-time at hand: the initial

[14] Quantization in static non-hyperbolic space-times was recently discussed in a series of papers [32]. The rigorous results obtained in those papers do not apply however to our situation.

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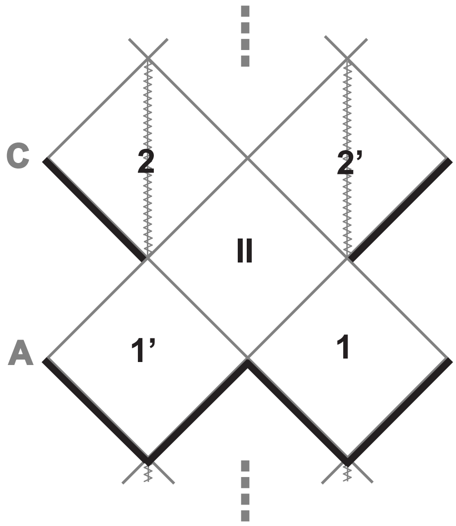

Figure 4: Initial data surface for the extended RN space-time.

data surface that sets up the first quantized Hilbert space, the canonical commutation relations and the way the commutation relations for the field modes can be determined. A state, analogous to the Hartle-Hawking vacuum, can be specified by requiring its invariance under the isometry (generated by ξ (2.42)) and the regularity of the stress-energy tensor expectation value (related to the Hadamard condition [32]). Correlators in such a state will be thermal with temperatures T_A and T_C for operators inserted only in the region A or C, respectively. Such a state, if it exists, has to be annihilated by certain operator modes. We construct two such operator modes whose commutator, computed with the help of canonical commutator relations, is a non-vanishing c number. This will lead us to the conclusion that a state that would be analogous, in the sense described above, to the Hartle-Hawking vacuum for the whole extended space-time at hand does not exist.

To define the first quantized Hilbert space of wave functions one can choose to specify the initial data for the Klein-Gordon equation on the asymptotic null-surfaces that have no causal past. This singles out the null-lines boldfaced in figure 4. We will assume that the corresponding null initial value problem is mathematically well defined.[15] We are thus exploring the possibility that a general pure state in this extended Hilbert space is of the form

$$|\Psi\rangle = \sum_{ijkl} C_{ijkl} |i\rangle_2 |j\rangle_{1'} |k\rangle_1 |l\rangle_{2'} \,, \tag{3.1}$$

where, for each $\lambda \in \{1, 1', 2, 2'\}$, the states $|j\rangle_\lambda$ form a complete basis for wave functions supported on the part of the initial data surface lying in region λ and vanishing on the rest of the initial data surface (see figure 4).

[15]One of the concerns on the mathematical side is what condition substitutes the usual smoothness assumption of the initial data when working with disconnected initial data surfaces as in the case at hand. Some questions related to the initial value problem in the region containing a singularity were considered in [5], where in particular it was shown that for the initial data specified on the past null infinities there is a unique solution to the wave equation defined on both sides of the singularity.

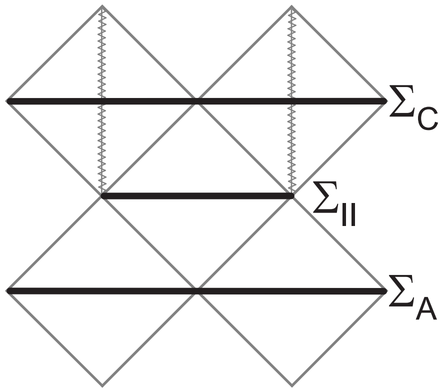

Figure 5: The fixed-time slices Σ_A, Σ_C and Σ_{II}.

Let $f_{\omega,l}$ be a complete basis to the space of solutions of the massless Klein-Gordon equation in the extended region of space-time at hand (with their initial data specified on the disconnected Cauchy surface of figure 4). The functions in this basis are labelled by the Killing vector eigenvalue ω, and an additional parameter l that takes finitely many values. In our discussion we are not going to use any specific basis. In appendix B we explain how to construct a basis of wave functions which are analytic with respect to the affine parameters on the horizons.

A quantum field in the extended space-time can be expanded as

$$\phi(x) = \sum_l \int_0^\infty d\omega\, a_{\omega,l} f_{\omega,l} + \text{h.c.}\,, \tag{3.2}$$

where $a_{\omega,l}$ are operators whose commutation relations are to be determined. The canonically conjugated momentum is an operator $\Pi(x,t) = \sqrt{-g}g^{tt}\partial_t\phi(x,t)$ that should satisfy the canonical equal time commutation relation

$$[\phi(x_1,t), \Pi(x_2,t)] = i\delta(x_1 - x_2)\,. \tag{3.3}$$

Also, we require that the standard causality assumption holds by which a commutator of the quantum field $\phi(x)$ with itself vanishes for space-like separated points.

Let us further introduce three fixed-time slices $\Sigma_A, \Sigma_C, \Sigma_{II}$ depicted in figure 5. Denote

$$(f,g)_{\Sigma_\delta} = i\int_{\Sigma_\delta} dx g^{tt}\sqrt{-g}\left(\bar{f}(x,t)\partial_t g(x,t) - g(x,t)\partial_t\bar{f}(x,t)\right) \tag{3.4}$$

— the Klein-Gordon inner products evaluated on the corresponding slices $\delta \in \{A, II, C\}$. By canonical commutation relations (3.3) and local causality we have on each slice Σ_δ:

$$[(\phi,f)_{\Sigma_\delta}, (\phi,\bar{g})_{\Sigma_\delta}] = (g,f)_{\Sigma_\delta}\,. \tag{3.5}$$

Noting that a Killing vector eigenmode is fixed by its eigenvalue ω and its value on the slices Σ_A and Σ_C, we see that substituting the expansion (3.2) into (3.5) for each slice Σ_δ allows one, in principle, to express commutation relations between all of the mode operators $a_{\omega,l}$, $a^\dagger_{\omega,l}$ in terms of the inner products $(f_{\omega,l}, f_{\omega',l'})_{\Sigma_A}$ and $(f_{\omega,l}, f_{\omega',l'})_{\Sigma_C}$, whose explicit form depends on the choice of basis at hand.

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In our discussion we rely on theorems proved by Kay and Wald [33] regarding the uniqueness of the Hartle-Hawking state in a globally hyperbolic static space-time with bifurcated horizons. In the course of the proof an algebraic state is defined as a mapping from an algebra of observables to complex numbers, satisfying the necessary properties of an average value (see [31] for details). In particular, such a definition of a state includes pure and mixed states realized in the standard approach to second quantization via the Fock space construction. The results of Kay and Wald apply to globally hyperbolic static regions with a (single) bifurcated Killing horizon. It is shown in [33] that if an algebraic state is invariant under the isometry (generated by a time-like Killing vector) and if the expectation value of the stress-energy tensor in this state is regular on the horizon, then this state is a pure state and it is unique. If furthermore there is a discrete isometry relating the static patches on the two sides of the horizon, a restriction of the state to one of the patches is thermal. It is called the Hartle-Hawking state; we denote the corresponding Hartle-Hawking state for regions A and C by $|\widetilde{HH}\rangle_A$ and $|\widetilde{HH}\rangle_C$, respectively. Given the space of classical solutions with initial data on a Cauchy surface and a symplectic structure one can define a field theory Hilbert space via specifying positive and negative frequency wave functions [31]. The Fock vacuum is a state which is annihilated by all the positive frequency modes. The Hartle-Hawking (pure) state is the Fock vacuum where positive and negative frequencies are defined with respect to the affine parameters along the (single) bifurcate Killing horizon. Here we describe an attempt to find an analog to the Hartle-Hawking state $|\widetilde{HH}\rangle$ for the extended CBH background. We start from a general pure state[16] in an extended Hilbert space

$$|\widetilde{HH}\rangle = \prod_i \left(\sum_{n_i^{(2)},n_i^{(1')},n_i^{(1)},n_i^{(2')}} C(n_i^{(2)},n_i^{(1')},n_i^{(1)},n_i^{(2')})|n_i^{(2)}\rangle_2|n_i^{(1')}\rangle_{1'}|n_i^{(1)}\rangle_1|n_i^{(2')}\rangle_{2'} \right) \tag{3.6}$$

where $|n_i^{(2)}\rangle_2|n_i^{(1')}\rangle_{1'}|n_i^{(1)}\rangle_1|n_i^{(2')}\rangle_{2'}$ stands for a state in the Fock space with $n_i^{(\lambda)} \in \mathbb{N}$ excited modes $|i\rangle_\lambda$. Restricting the state $|\widetilde{HH}\rangle$ to the regions of type A (C) of figure 4 one obtains an algebraic state in these regions. We mark this *algebraic* state by $\widetilde{HH}_A$ ($\widetilde{HH}_C$). If $|\widetilde{HH}\rangle$ is invariant under the isometry and its expected stress-energy tensor is regular on the horizons, so is $\widetilde{HH}_A$ ($\widetilde{HH}_C$). Note that regions A and C are static and globally hyperbolic (for the class of initial conditions in region C obtained by setting up the data on the past null infinities [5]). Thus, according to [33], $\widetilde{HH}_A$ ($\widetilde{HH}_C$) have to be equal to $|\widetilde{HH}\rangle_A$ ($|\widetilde{HH}\rangle_C$). Strictly speaking the theorems of Kay and Wald may not be applicable for the region C which has singularities. However, as our first concern is the behavior of the stress-energy tensor on the horizons we may restrict our considerations to a globally hyperbolic subregion in between the singularities that contains the bifurcation point of the horizon. In this subregion the results of Kay and Wald are directly applicable. This means that the restriction of $|\widetilde{HH}\rangle$ to such a subregion $\widetilde{C}$ will be equal to the corresponding Hartle-Hawking state $|\widetilde{HH}\rangle_{\widetilde{C}}$. In what follows we will not restrict to such a subregion although all our arguments apply for such a restriction.

[16]This assumption is made only for the sake of simplifying the argument; it can be also shown to hold for a mixed state.

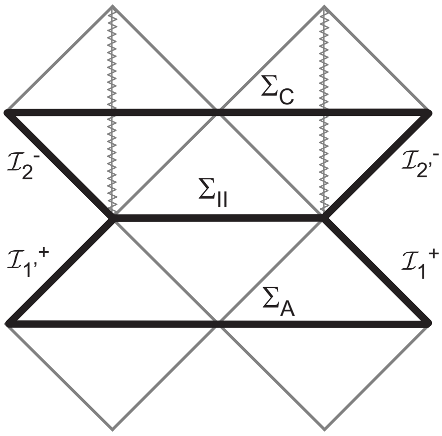

Figure 6: The null surfaces $\mathcal{I}_1^+$, $\mathcal{I}_{1'}^+$ stand for asymptotic future null-infinities of regions 1 and $1'$, respectively, and $\mathcal{I}_2^-$, $\mathcal{I}_{2'}^-$ denote the asymptotic past null-infinities of regions 2 and $2'$.

We see that if we consider a restriction of the field $\phi(x)$ to region A its positive frequency components $(\phi, g_\omega)_{\Sigma_A}$, defined with respect to affine times on the outer horizon, have to annihilate $|\widetilde{HH}\rangle$. Here g_ω is an arbitrary Killing vector eigenmode that contains only positive frequency modes with respect to affine times on the outer horizon. Analogously, the operators $(\phi, h_\omega)_{\Sigma_C}$ have to annihilate $|\widetilde{HH}\rangle$ for any function h_ω comprised out of positive frequencies which are defined only with respect to affine times on the inner horizon. We are going to demonstrate that these conditions are not mutually compatible by constructing two such annihilation operators that commute to a c number. To that end let us construct a specific mode $F_\omega(x)$ that has the property that it consists of pure positive affine time frequencies in region A and of pure negative affine time frequencies in region C (the affine times being defined on the outer and inner horizons, respectively).

The state $|\widetilde{HH}\rangle$ then has to be annihilated by operators $(\phi, F_\omega)_{\Sigma_A}$, $(\phi, \bar{F}_\omega)_{\Sigma_C}$ where $\bar{F}_\omega$ stands for the complex conjugate function and hence contains only positive frequency modes with respect to affine times on the inner horizon. The commutator of these two operators can be computed as follows.

First note that by Klein-Gordon norm conservation we have

$$\begin{aligned}(\phi, F_\omega)_{\Sigma_A} &= (\phi, F_\omega)_{\mathcal{I}_1^+} + (\phi, F_\omega)_{\Sigma_{II}} + (\phi, F_\omega)_{\mathcal{I}_{1'}^+} \\ (\phi, \bar{F}_\omega)_{\Sigma_C} &= (\phi, \bar{F}_\omega)_{\mathcal{I}_2^-} + (\phi, \bar{F}_\omega)_{\Sigma_{II}} + (\phi, \bar{F}_\omega)_{\mathcal{I}_{2'}^-}\end{aligned} \tag{3.7}$$

and analogous expressions for other operators. Here $\mathcal{I}_1^+$, $\mathcal{I}_{1'}^+$ stand for asymptotic future null-infinities of regions 1 and $1'$, respectively, and $\mathcal{I}_2^-$, $\mathcal{I}_{2'}^-$ denote the asymptotic past null-infinities of regions 2 and $2'$; see figure 6. Since the regions $\mathcal{I}_1^+$, $\mathcal{I}_{1'}^+$ are causally disconnected and disjoint from the regions $\mathcal{I}_2^-$, $\mathcal{I}_{2'}^-$ the corresponding restrictions of operators $\phi(x,t)$ and $\Pi(x,t)$ commute. Therefore, by (3.5) and (3.7) we have

$$[(\phi, F_\omega)_{\Sigma_A}, (\phi, \bar{F}_\omega)_{\Sigma_C}] = (F_\omega, F_\omega)_{\Sigma_{II}}\,. \tag{3.8}$$

The mode F_ω can be chosen to be any arbitrary Killing vector eigenmode in region II, in particular it can be chosen in such a way that $(F_\omega, F_\omega)_{\Sigma_{II}} \neq 0$ (an example of such a

Killing eigenmode is the one which vanishes on the outer horizon of region 1). We conclude that the quantity on the right hand side of (3.8) is a c number which generically does not vanish. This presents an obstruction to the existence of an analog to the Hartle-Hawking vacuum (3.6) because there cannot be a state simultaneously annihilated by the operators $(\phi, F_\omega)_{\Sigma_A}$ and $(\phi, \bar{F}_\omega)_{\Sigma_C}$.[17]

Finally, to build such a function as F_ω we start with an arbitrary Killing vector eigenmode F_ω^{II} in region II. The values of F_ω^{II} are thus specified on the future part of the outer horizon and on the past part of the inner horizon. Consider the restrictions $F_\omega^{II}(U_{\rm out})$, $F_\omega^{II}(V_{\rm out})$ to the future half of the outer horizon $U_{\rm out} > 0$, $V_{\rm out} > 0$, where $U_{\rm out}$, $V_{\rm out}$ are the affine parameters. The positive frequency extensions of F_ω^{II} to the past outer horizon then read [34]

$$F_\omega(U_{\rm out}) = \begin{cases} F_\omega^{II}(U_{\rm out}) & U_{\rm out} > 0 \\ e^{\pi\omega} F_\omega^{II}(-U_{\rm out}) & U_{\rm out} < 0 \end{cases} \tag{3.9}$$

and

$$F_\omega(V_{\rm out}) = \begin{cases} F_\omega^{II}(V_{\rm out}) & V_{\rm out} > 0 \\ e^{\pi\omega} F_\omega^{II}(-V_{\rm out}) & V_{\rm out} < 0\,. \end{cases} \tag{3.10}$$

Once the function is specified on all parts of the outer horizon it has a unique extension to region A. Analogously, we extend F_ω^{II} to region C, but this time to consist of only negative modes with respect to the affine parameters $U_{\rm in}$, $V_{\rm in}$ on the inner horizon.

3.2 Quantization 2

The second approach to quantization of a scalar field in the 2-d CBH background is based on the $\mathrm{SL}(2,\mathbb{R})$ structure underlying the model at hand and is more inspired by the underlying CFT and string theory. Every mode of the scalar field corresponds to a low lying vertex operator in string theory. Vertex operators in $\frac{\widetilde{\mathrm{SL}}(2,\mathbb{R})\times \mathrm{U}(1)}{\mathrm{U}(1)}$ are vertex operators in $\widetilde{\mathrm{SL}}(2,\mathbb{R})\times \mathrm{U}(1) = AdS_3 \times \mathrm{U}(1)$ that are invariant under the gauge transformation (2.17). In string theory we associate different space-time fields with different modes of the string (vertex operators) that do not mix under the global symmetry group (which is the space-time isometry group). Thus each space-time field is characterized by a unitary representation of the global symmetry group. The global symmetry group of the string moving on a group manifold is the group itself, in our case it is $\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{U}(1)$. After doing a KK reduction along the $\mathrm{U}(1)$, gauge invariant vertex operators in $\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{U}(1)$ with different momentum along the $\mathrm{U}(1)$ factor give different KK particles. Here however we are interested only in the low lying excitations of the KK tower. Thus we are interested in the unitary representations of the group $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ and the corresponding wave functions which we now review.

Any representation of $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ is characterized by two Casimir eigenvalues j and ϵ. The Casimir eigenvalue ϵ takes values in the range $[0,1/2]$ and describes how the states change

[17] It should be emphasized that the conclusion is drawn in the framework of semiclassical quantization that does not take significantly into account any stringy effects.

when we go from one region to another in $\widetilde{\rm SL}(2,\mathbb{R})$.[18] The second Casimir eigenvalue j is related to the laplacian eigenvalue $c_2 = -J_3^2 + J_1^2 + J_2^2$ as $c_2 = -j(j+1)$ (where J_1, J_2 and J_3 are the generators of ${\rm SL}(2,\mathbb{R})$). For each representation ρ of $\widetilde{\rm SL}(2,\mathbb{R})$ one can define a wave function on the group manifold as $f(g) = \langle\psi|\rho(g)|\psi'\rangle$ where $g \in \widetilde{\rm SL}(2,\mathbb{R})$. A complete basis of functions in $\mathcal{L}^2\left(\widetilde{\rm SL}(2,\mathbb{R})\right)$ is composed in this way out of two types of unitary representations. These two types are discrete representations, for which $j > -1/2$ take real values, and continuous representations, for which $j = -1/2 + is,\ s \in \mathbb{R}$. In the discrete representations the wave function falls exponentially at infinity and describes a normalized excitation of the field which lives far from the boundary. In the continuous representations the wave function is delta function normalizable and describes a scattering mode. While j has to do with a local behavior of the wave function, ϵ is a global characteristic and plays a role only when we go between regions in $\widetilde{\rm SL}(2,\mathbb{R})$.

When we gauge a subgroup inside $\widetilde{\rm SL}(2,\mathbb{R})$ only a gauge invariant subspace of wave functions for each representation gives the wave functions on the coset.[19] Note that given two waves with the same j, an observer that sits in a single region cannot tell if the waves are of the same type (have the same ϵ) or of different types (have different ϵ's).

After going to the CBH coset, from the point of view of quantization 1, described in the previous subsection, picking a specific ϵ corresponds to picking a specific one-to-one correlation between the initial data on Σ_A and the initial data on $\mathcal{I}_2^-$, $\mathcal{I}_{2'}^-$ (see figure 6).[20] A mode characterized by a fixed ϵ that is made from purely positive frequencies with respect to the affine parameters along the outer horizon is necessarily made from a mix of positive and negative frequencies with respect to the affine parameters along the inner horizon (see appendix C for details in the case $\epsilon = 0$). We thus conclude that in the second approach to quantization for a given species of waves (characterized by a fixed ϵ) there is no state in which one has thermal distributions in regions of type A and C with the corresponding temperatures — a would be analog of the Hartle-Hawking vacuum for the whole extended space-time at hand does not exist.

3.3 Conclusions

In [33] Kay and Wald proved that the Hartle-Hawking vacuum is the unique vacua which is invariant under the isometries and in which the expectation value of the stress-energy tensor is regular on the horizon. This leads us to the conclusion that in any Fock vacuum of the extended CBH, which is invariant under the isometries, the expectation value of the stress-energy tensor diverges at least on one of the horizons. The divergence of the stress-energy tensor at a smooth point in space-time means that the energy density there is large and a back reaction has to be taken into account already in a vacuum state. The singularity is not a smooth part of the background. If a proper analog of the Hartle-Hawking vacuum $|\widetilde{HH}\rangle$ with a non-singular stress-energy tensor on both of the horizons existed, we still would

[18]More precisely, if we represent points $g \in \widetilde{\rm SL}(2,\mathbb{R})$ as we did in (2.5), (2.20) then ϵ describes how the states change from the point g to $g \cdot i\sigma_2$ (which is a step along the compact direction of ${\rm SL}(2,\mathbb{R})$ that is unwrapped in $\widetilde{\rm SL}(2,\mathbb{R})$).

[19]For generic charge the discrete representations do not survive the euclidean gauging.

[20]In that case the operators $\phi(x,t)$ and $\Pi(x,t)$ restricted to $\mathcal{I}_1^+$, $\mathcal{I}_{1'}^+$ and to $\mathcal{I}_2^-$, $\mathcal{I}_{2'}^-$ do not commute.

have to study the behavior of the stress-energy tensor on the singularities. However, as we showed above, a simple would be Hartle-Hawking state does not exist and the stress-energy tensor is already singular at one of the horizons.

A possible interpretation of this result is the following. Even in a ground state, particle creation beyond the event horizon leads to a large back reaction causing a divergent stress-energy tensor at the inner horizon.

4. Cosmology with whiskers

4.1 Lorentzian Nappi-Witten solution — a review

A coset CFT which is closely related to the 2-d CBH is the cosmological Nappi-Witten background [16]. Recently, it was studied in more detail in [3, 4]. This background is obtained from a coset CFT:

$$\left(\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{SU}(2)\right) / \left(\mathrm{U}(1) \times \mathrm{U}(1)\right) \,. \tag{4.1}$$

Let $(g_1, g_2) \in \widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{SU}(2)$. The non-anomalous $\mathrm{U}(1) \times \mathrm{U}(1)$ group action is chosen as

$$\begin{aligned} \delta g_1 &= \epsilon\sigma_3 g_1 + (\tilde{\epsilon}\cos(\alpha) - \epsilon\sin(\alpha)) g_1 \sigma_3 \,, \\ \delta g_2 &= i\tilde{\epsilon}\sigma_3 g_2 + (\tilde{\epsilon}\sin(\alpha) + \epsilon\cos(\alpha)) g_2 i\sigma_3 \,, \end{aligned} \tag{4.2}$$

where α is an angular variable analogous to the parameter ψ that labels the CBH backgrounds. The four-dimensional space-time manifold described by the corresponding gauged WZW contains six regions depicted in figure 7 (see [3] for details), which are cyclically repeated in the maximally extended solution. Two out of these six regions are time-dependent and describe a universe evolving from a big bang to a big crunch singularity (these regions are marked by the letter C in figure 7). The remaining four regions are static, contain closed time-like curves and are usually referred to as "whiskers" [3] (marked by the letter W in figure 7). In addition to the presence of closed time-like curves the whiskers contain a time-like singularity surface called a "domain wall" in [3]. Explicitly the background in a whisker is described as follows. Let us choose a parameterization of the corresponding submanifold of $\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{SU}(2)$ as

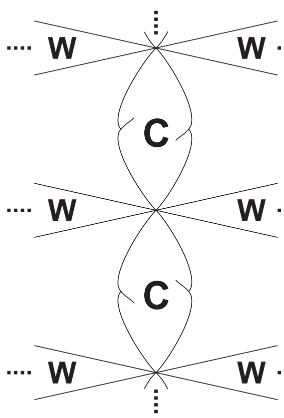

Figure 7: Nappi-Witten Cosmology.

$$\begin{aligned} g_1 &= e^{\gamma\sigma_3} e^{\theta\sigma_1} e^{\beta\sigma_3} \,, \\ g_2 &= e^{i\gamma'\sigma_3} e^{i\theta'\sigma_2} e^{i\beta'\sigma_3} \end{aligned} \tag{4.3}$$

(this corresponds to the $\epsilon = \epsilon' = 0$, $\delta = I$ region in the notation of [3]). Here $g_1 \in \widetilde{\mathrm{SL}}(2,\mathbb{R})$, $g_2 \in \mathrm{SU}(2)$. We choose the gauge fixing condition $\gamma = \beta = 0$. There are no residual gauge transformations in this case since γ and β are noncompact. The metric in such a whisker can be derived to be

$$\begin{aligned} \frac{ds^2}{k} &= (d\theta)^2 + (d\theta')^2 + g_{\lambda_+\lambda_+}(d\lambda_+)^2 + g_{\lambda_-\lambda_-}(d\lambda_-)^2 \,, \qquad (4.4)\\ g_{\lambda_-\lambda_-} &= -\frac{\tanh^2(\theta)}{b^2 - \tanh^2(\theta)\cot^2(\theta')} \,,\\ g_{\lambda_+\lambda_+} &= \frac{b^2\cot^2(\theta')}{b^2 - \tanh^2(\theta)\cot^2(\theta')} \,,\\ b^2 &= \frac{1-\sin(\alpha)}{1+\sin(\alpha)} \,. \qquad (4.5) \end{aligned}$$

Here $\lambda_\pm = \gamma' \pm \beta'$ have periodicity of 2π. As evident from the form of the metric, shifts of the coordinates $\lambda_\pm$ generate two commuting isometries.

In addition there are nontrivial B-field and dilaton backgrounds

$$\begin{aligned} B_{\lambda_+\lambda_-} &= \frac{kb^2}{b^2 - \tanh^2(\theta)\cot^2(\theta')} \,,\\ \Phi &= \Phi_0 - \frac{1}{2}\log(\cosh^2(\theta)\sin^2(\theta') - b^2\sinh^2(\theta)\cos^2(\theta')) \,. \qquad (4.6) \end{aligned}$$

The surface specified by the equation

$$b^2 - \tanh^2(\theta)\cot^2(\theta') = 0$$

is a curvature singularity to which we refer to as a singular domain wall.

4.2 Euclidean Nappi-Witten background

The euclidean Nappi-Witten background is obtained as follows. Start with the $\widetilde{\mathrm{SL}}(2,\mathbb{R}) \times \mathrm{SU}(2)$ WZW model and gauge away one time-like and one space-like U(1) so that the remaining space is a four-dimensional euclidean one. A general non-anomalous $\widetilde{U}(1) \times \mathrm{U}(1)$ action of this kind has the form

$$\begin{aligned} \delta g_1 &= \epsilon i\sigma_2 g_1 + (\tilde{\epsilon}\eta_1\eta_2\sinh(\alpha_E) + \epsilon\eta_1\cosh(\alpha_E))g_1 i\sigma_2 \,,\\ \delta g_2 &= i\tilde{\epsilon}\sigma_3 g_2 + (\tilde{\epsilon}\eta_2\cosh(\alpha_E) + \epsilon\sinh(\alpha_E))g_2 i\sigma_3 \,. \qquad (4.7) \end{aligned}$$

Here $\eta_1, \eta_2 = \pm 1$ are discrete parameters corresponding to the axial/vector choices of gaugings on $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ and SU(2), respectively.

Let us concentrate on the case where $\eta_1 = 1$, $\eta_2 = -1$.[21] We choose parameterizations

$$\begin{aligned} g_1 &= e^{i\gamma\sigma_2}e^{\theta\sigma_3}e^{i\beta\sigma_2} \,,\\ g_2 &= e^{i\gamma'\sigma_3}e^{i\theta'\sigma_2}e^{i\beta'\sigma_3} \,. \qquad (4.8) \end{aligned}$$

Here $(\gamma + \beta)$ is the compact time in $\mathrm{SL}(2,\mathbb{R})$ that should be unwrapped to obtain the universal cover $\widetilde{\mathrm{SL}}(2,\mathbb{R})$. We can perform an initial gauge fixing by imposing the condition

[21] The signs of the η's are interchanged by T-duality.

$\gamma = \beta = 0$. We obtain the following metric, B-field and dilaton:

$$
\begin{aligned}
\frac{ds^2}{k} &= (d\theta)^2 + (d\theta')^2 + \frac{\tanh^2(\theta)(d\lambda_-)^2}{b_E^2 + \tanh^2(\theta)\cot^2(\theta')} + \frac{b_E^2\cot^2(\theta')(d\lambda_+)^2}{b_E^2 + \tanh^2(\theta)\cot^2(\theta')}\,, \\
B_{\lambda_+\lambda_-} &= \frac{kb_E^2}{b_E^2 + \tanh^2(\theta)\cot^2(\theta')}\,, \\
\Phi &= \Phi_0 - \frac{1}{2}\log(\cosh^2(\theta)\sin^2(\theta') + b_E^2\sinh^2(\theta)\cos^2(\theta'))\,,
\end{aligned} \tag{4.9}
$$

where

$$
b_E^2 = \coth^2\left(\frac{\alpha_E}{2}\right), \tag{4.10}
$$

and as in (4.4) $\lambda_\pm = \gamma' \pm \beta'$ both have periodicity of 2π, but now in contrast with the Minkowski signature case, the coordinate $\gamma - \beta$ is compact. It is easy to derive from (4.7) that the residual gauge transformations preserving $\gamma + \beta$ and shifting $\gamma - \beta$ by an integer multiple of 2π are generated by the shift

$$
\lambda_- \to \lambda_- + 2\pi b_E\,. \tag{4.11}
$$

Comparing (4.9) with (4.4) we see that the domain wall has disappeared. The subspace $\theta = \theta' = 0$ is singular, it has a trumpet-like curvature singularity. There are also potential conical singularities at $\theta = 0$, $\theta' = 0$ and $\theta' = \pi/2$ subspaces when the coordinate λ_- or λ_+ shrinks to zero size. The identifications $\lambda_\pm \sim \lambda_\pm + 2\pi$ and (4.11) are precisely those needed to avoid those conical singularities. Since the projection with respect to the transformations (4.11) happens in addition to the original periodicity of λ_- of 2π we see that unless the parameter b_E is rational the coordinate λ_- is completely degenerate.

Another way to see the same effect is by studying the spectrum. An irreducible representation of the universal cover of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ is labelled by numbers $m, \bar{m}, j$, $m', \bar{m}', j'$. Even on the universal cover the numbers $m, \bar{m}$ must satisfy $m - \bar{m} \in \mathbb{Z}$. The gauge invariance condition reads

$$
\begin{aligned}
m + \bar{m}\cosh(\alpha_E) + \bar{m}'\sinh(\alpha_E) &= 0\,, \\
m' - \bar{m}\sinh(\alpha_E) - \bar{m}'\cosh(\alpha_E) &= 0\,.
\end{aligned} \tag{4.12}
$$

From these formulas it is easy to derive

$$
m - \bar{m} = \frac{1 + \cosh(\alpha_E)}{\sinh(\alpha_E)}(\bar{m}' - m')\,.
$$

Since $\bar{m}' - m'$ is an integer we see that $m - \bar{m}$ can be a nonzero integer only if

$$
\frac{1 + \cosh(\alpha_E)}{\sinh(\alpha_E)} = \coth\left(\frac{\alpha_E}{2}\right) = b_E
$$

is a rational number. Generically however we have only the zero solution $m = \bar{m} = 0$ that describes only zero momentum states.

The nature of this phenomenon can be tracked down to having a situation when a compact torus is quotiented by a U(1) whose position is incommensurate with the torus lattice. The corresponding gauge orbit is everywhere dense on the torus and we are left with no momenta (except zero) on the quotient.

In the case when b_E is rational the corresponding coset space is a four-dimensional euclidean manifold that can be described as follows. Assume that $b_E = P/Q$ where P, Q are relatively prime. Then it follows from (4.11) that λ_- has a radius of $1/Q$ and λ_-/b_E has a radius of $1/P$. This implies then that the metric (4.9) has an orbifold type conical singularities respectively at $\theta' = 0$ of a deficit angle $2\pi/Q$ and at $\theta = 0$ of a deficit angle $2\pi/P$. Thus the only nonsingular case is $|b_E| = 1$ (in that case gauging $\mathrm{SL}(2,\mathbb{R})$ instead of $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ would not add a new identification and we would get the same euclidean geometry).

In parallel with our discussion of euclidean CBH background the euclidean space (4.9) is obtained from the lorentzian one (4.4) by the following Wick rotation:

$$\alpha \to i\alpha_E - \frac{\pi}{2}, \tag{4.13}$$

where we included a shift by $-\pi/2$ to match the conventions of (4.5) with (4.10) (which reads $b^2 \to -b_E^2$). From the point of view of the Wick rotated background the periodicities of $\lambda_\pm$ are imposed to avoid conical singularities at $\theta = 0$, $\theta' = 0$ and $\theta' = \pi/2$. As in the above discussion of the euclidean coset, unless $|b_E| = 1$, the required identifications yield a degenerate, effectively three-dimensional spectrum.

In the asymptotic region $\theta \to \infty$ the metric (4.9) takes the form

$$\frac{ds^2}{k} \approx (d\theta)^2 + (d\theta')^2 + \frac{(d\lambda_-)^2}{b_E^2 + \cot^2(\theta')} + \frac{b_E^2 \cot^2(\theta')(d\lambda_+)^2}{b_E^2 + \cot^2(\theta')},$$

which is uniformly bounded in both $\lambda_\pm$ directions. The maximal asymptotic radius of the λ_- direction is b_E^{-2} and is achieved at $\theta' = \pi/2$ while the maximal asymptotic radius of the λ_+ direction is b_E^2 which is achieved at $\theta' = 0$. The maximal radii are equal when the conical singularities are absent that is when $b_E^2 = 1$. In the Minkowski signature space one of the Killing vectors is time-like. Since in the euclidean solution the asymptotic values of both λ_+ and λ_- (euclidean) directions are bounded this suggests that at $b_E^2 = 1$ the euclidean solution defines a vacuum state in the original Minkowski signature space that is thermal. The dependence of the asymptotic sizes on θ' presumably can be interpreted in terms of the high anisotropy of the outgoing thermal radiation. In the euclidean space both Killing directions are completely on equal footing. Thus it seems likely to us that the euclidean solution defines a vacuum characterized by a canonical distribution in the eigenvalues of both Killing vectors $\frac{\partial}{\partial\lambda_\pm}$.

In the above discussion of the euclidean background we concentrated on a coset (4.7) with $\eta_1 = 1$, $\eta_2 = -1$, which is related by the Wick rotation (4.13) to the Minkowski signature whisker characterized in the notation of [3] by the values $\epsilon = \epsilon' = 0$, $\delta = I$. Each of the pairs of whiskers (1, 3) and (1', 3') is described by the same euclidean background. The whiskers 1 and 1' are related by interchanging λ_+ with λ_- (the same goes for 3 and 3'). The Wick rotated background in whiskers 2 and 4 (obtained by the same Wick rotation as in (4.13)) can also be obtained by considering the euclidean coset (4.7), corresponding to the choice $\eta_1 = -1$, $\eta_2 = 1$. The two euclidean cosets are T-dual to each other. The corresponding metric, B-field and dilaton are obtained from the ones in (4.9) by the change $\theta \to i\pi/2 - \theta$. The background in 2' (4') is obtained by the interchange mentioned above. The residual gauge transformations for the T-dual euclidean coset are now generated by the shift

$$\lambda_+ \to \lambda_+ + 2\pi b_E^{-1} \,.$$

The two types of the euclidean whiskers obtained have the same singularity structure outlined before. Thus, for irrational b_E they both degenerate. The conical singularities are absent only when $|b_E| = 1$ and the trumpet-like curvature singularity is present for both types. The backgrounds in the regions 1 and 2 (as well as 3, 4 and in the corresponding primed whiskers) are different for a general value of the variable θ. However, for large values of that variable and $|b_E| = 1$ they have the same form. This is unlike the generic case of the CBH, where the two euclidean backgrounds have also two different asymptotic temperatures. It is more like the case of the extremal CBH.

To summarize, we see that in distinction with the CBH case the euclidean coset CFT's that are related to the Minkowski signature CFT via Wick rotation demonstrate a certain degeneracy or non-smoothness for generic values of the euclidean mixing angle parameter b_E. For irrational values of b_E one coordinate is degenerate and, classically, the euclidean background degenerates to a three-dimensional one. For a rational $b_E \neq \pm 1$ the resulting background is four-dimensional but possesses orbifold-type conical singularities, leading to a degenerate spectrum. This situation is very much reminiscent of the situation with euclidean Milne Universe [35]. The corresponding space also generically has a conical singularity and its spectrum is truncated to zero modes for generic values of a certain angular parameter that is analogous to our b_E. In addition to those features the euclidean backgrounds for all values of b_E also contain a trumpet-like singularity. The last one can be potentially removed by the method developed in [4]. We leave this question to a future investigation.

Acknowledgments

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A. Euclidean Kerr black hole

For completeness, we show how one can compute the chemical potential Ω_H conjugate to angular momentum in a Kerr black hole, in an analogous way to the euclidean computation of $\mu_{\rm el}$ for RN.

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To make the analog more clear we first start by noting that the 2-d CBH is actually a Kaluza-Klein (KK) reduction of a 3-d dilatonic rotating black string, where the 2-d gauge field is related to the black string angular momentum. In what follows we will show how the condition that A_0 vanishes at the tip of the euclidean CBH (ECBH) comes from the regularity condition of the euclidean black string. We then show that the condition for the regularity of the euclidean black string is the same as the one for the euclidean Kerr black hole, and read Ω_H from the regular Kerr metric.

The 3-d black string metric in the region of the black string which after the KK reduction becomes region A of the CBH is given by [2]:

$$ds^2 = d\theta^2 - \frac{dy^2 - 2pdydx}{\coth^2(\theta) - p^2} + dx^2\,, \tag{A.1}$$

where $x = x + 2\pi L$. After the KK reduction the 2-d metric, dilaton and gauge field become as in (2.6), (2.9). After euclidean continuation $y \to i\tau$ and $p \to -ip$, (A.1) becomes[22]

$$ds^2 = d\theta^2 + \frac{d\tau^2 + 2pd\tau dx}{\coth^2(\theta) + p^2} + dx^2\,. \tag{A.2}$$

Near the horizon $\theta = 0$ of the black string the metric (A.1) is

$$ds^2 \sim d\theta^2 + \left(dx + p\theta^2 d\tau\right)^2 + \theta^2 d\tau^2\,. \tag{A.3}$$

Upon coordinate transformation $\tilde{x} = x - c\tau$ (which is equivalent to a large gauge transformation in the lorentzian CBH), the same metric is

$$ds^2 \sim d\theta^2 + \left(d\tilde{x} + (c + p\theta^2)d\tau\right)^2 + \theta^2 d\tau^2\,. \tag{A.4}$$

Now, to have a non-singular metric we compactify the τ direction $(x,\tau) = (x,\tau + 2\pi)$. If instead we identify $(\tilde{x},\tau) = (\tilde{x},\tau + 2\pi)$ then at $\theta = 0$ we are identifying $x = x + 2\pi L$ and $x = x + 2\pi c$, where the metric is $ds^2 = d\theta^2 + dx^2$. If L/c is not rational then the geometry is singular at $\theta = 0$. The same singularity is seen in the ECBH. A KK reduction along the $\tilde{x}$ direction leads to a singular gauge field at $\theta = 0$, unless all the charges are quantized in units of $1/c$. Upon the KK reduction along the x direction, we get a regular gauge field which supports a non-trivial Wilson loop at infinity. The chemical potential is read from this asymptotic Wilson loop. In a Kerr black hole the situation is the same.

The Kerr metric in four dimensions is given by:

$$ds^2 = \frac{\rho^2}{r^2 f}dr^2 - \frac{r^2 f}{\rho^2}\left[dt - a\sin^2\theta d\phi\right]^2 + \rho^2 d\theta^2 + \frac{\sin^2}{\rho^2}\left[(r^2 + a^2)d\phi - adt\right]^2\,, \tag{A.5}$$

where

$$f = 1 - \frac{2M}{r} + \frac{a^2}{r^2}\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,. \tag{A.6}$$

The horizons are located at $(M > |a|)$:

$$r_\pm = M \pm \sqrt{M^2 - a^2}\,. \tag{A.7}$$

[22]Alternatively, this background is obtained from the euclidean coset (2.17).

The euclidean metric is obtained by taking $t \to i\tau$ and at the same time $a \to i a_E$:

$$ds_E^2 = \frac{\rho^2}{r^2 f} dr^2 + \frac{r^2 f}{\rho^2}\left[d\tau - a_E \sin^2\theta d\phi\right]^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2 - a_E^2) d\phi + a_E d\tau\right]^2 . \quad (A.8)$$

In terms of $\tilde{r} \equiv r - r_+$ the euclidean metric near the horizon becomes:

$$\begin{aligned} ds_E^2 \sim & \frac{r_+^2 - a_E^2}{\tilde{r}(r_+ - r_-)} d\tilde{r}^2 + \frac{\tilde{r}(r_+ - r_-)}{r_+^2 - a_E^2}\left[d\tau - a_E \sin^2\theta d\phi\right]^2 + \\ & + (r_+^2 - a_E^2) d\theta^2 + \sin^2\theta (r_+^2 - a_E^2)\left[d\phi + \Omega_H d\tau\right]^2 , \end{aligned} \quad (A.9)$$

where

$$\Omega_H = \frac{a_E}{r_+^2 - a_E^2} \quad (A.10)$$

is the angular velocity at the horizon. This euclidean geometry is singular at $\theta = 0$, $\tilde{r} = 0$ since at $\theta = 0$ (A.9) becomes

$$ds^2_{E_{|\theta=0}} \sim \frac{r_+^2 - a_E^2}{\tilde{r}(r_+ - r_-)} d\tilde{r}^2 + \frac{\tilde{r}(r_+ - r_-)}{r_+^2 - a_E^2} d\tau^2 . \quad (A.11)$$

To smooth out the geometry we compactify the euclidean time

$$\tau = \tau + \frac{2\pi}{\kappa} = \tau + 2\pi\beta , \quad (A.12)$$

where the surface gravity κ is given by

$$\kappa = \frac{1}{2}\left(\frac{\tilde{r}(r_+ - r_-)}{r_+^2 - a_E^2}\right)'_{|\tilde{r}=0} = \frac{r_+ - r_-}{2(r_+^2 - a_E^2)} . \quad (A.13)$$

Comparing (A.2) with (A.9) at $\tilde{r} = 0$ and $\theta \sim 0$, we see that in order to avoid a singularity at $\theta = 0$ we should add to (A.12) a shift in $\tilde{\phi} = \phi + \Omega_H \tau$ as follows:

$$(\tilde{\phi}, \tau) = (\tilde{\phi}, \tau + 2\pi\beta) . \quad (A.14)$$

Now asymptotically the metric (A.8) becomes

$$ds_E^2 \sim dr^2 + d\tau^2 + r^2 d\theta^2 + r^2 \sin^2\theta \left(d\tilde{\phi} - \Omega_H d\tau\right)^2 . \quad (A.15)$$

As in the 3-d black string case, upon the KK reduction along the $\tilde{\phi}$ coordinate we get a non-trivial Wilson loop at infinity from which we read off the chemical potential for charged particles $\mu_{\mathrm{el}} = \Omega_H$. Before the KK reduction this charged particles are particles with momentum along the $\tilde{\phi}$ direction which are particles with angular momentum. We learn that in a Kerr black hole the chemical potential conjugate to angular momentum is Ω_H, which is by now very well known.

B. Relations between modes in different regions

In this appendix we discuss how to extend wave functions from one region to a basis in the extended CBH background, while preserving analyticity with respect to the affine parameters on the horizons. This basis of wave functions can be used to bring correlators in any state between operators in different regions to a linear combination of correlators where all insertions are in the same region with some imaginary shifts. We start by reviewing a simpler example of an analogous construction in Rindler space. Rindler space is a flat Minkowski space-time as seen by an observer that travels with a constant acceleration. Although, as seen by the accelerated observer, there are parts of the space-time which are hidden behind horizons, there is noting special happening at those points physics-wise. A vacuum state (for example, the Minkowski vacuum) in which there is noting special at those points (the expectation value of the stress-energy tensor is everywhere smooth) is built out of modes which are locally analytic with respect to any locally flat coordinate system (for example, the affine parameter along the bifurcate horizons near the bifurcating pint). Rindler coordinates (r,η) run from $-\infty$ to ∞ and cover Minkowski space-time (x,t) in four patches (see figure 8),

$$x \pm t = \begin{cases} re^{\pm\eta} & R, L \\ \pm re^{\pm\eta} & F, P \end{cases}, \qquad ds^2 = dx^2 - dt^2 = \begin{cases} -r^2 d\eta^2 + dr^2 & R, L \\ r^2 d\eta^2 + dr^2 & F, P \end{cases} \tag{B.1}$$

where in regions R and F we have $r > 0$ and $r < 0$ in regions P and L. The region R is obtained from the region F by a transformation $(r,\eta) \to (\mp ir, \eta \pm i\frac{\pi}{2})$, where the line separating the two regions is invariant. In the same way the region L is obtained from the region F by $(r,\eta) \to (\pm ir, \eta \pm i\frac{\pi}{2})$. The modes which are locally analytic on the horizons have their values in the regions L and R obtained from their value in region F by the above transformations.

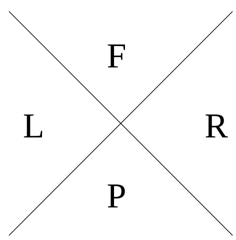

Figure 8: Rindler coordinate patches.

The situation on each bifurcate horizon of the CBH background is the same. Here we parameterize the regions in $SL(2,\mathbb{R})$ correspond to the compact regions of the CBH by $g_{II} = e^{\frac{1}{2}(z+y)\sigma_3} e^{i\theta\sigma_2} e^{\frac{1}{2}(z-y)\sigma_3}$ where $\theta \in [-\pi/2, 0]\,,\ [0,\pi/2]\,,\ [\pi/2,\pi]$ in regions I, II and III correspondingly. We fix the gauge by setting z to zero. In that parametrization the background fields of regionz I, II and III are obtained from (2.6) by taking $\theta \to i\theta$. The CBH metric in the (y,θ) coordinates (regions 1, 1', I, II) or $(y/p^2,\theta)$ coordinates (regions 2, 2', II, III) looks locally near the bifurcating points as a Rindler metric and the dilaton is approximately constant. The analog of the coordinates $x \pm t$ in the Rindler space are the affine parameters U, V along the horizons. The local coordinates (y,θ) are given in terms of the affine parameters by

$$V_A \pm U_A = \begin{cases} (e^y \mp e^{-y})\sinh(\theta) & 1, 1' \\ (e^y \pm e^{-y})\sin(\theta) & I, II \end{cases}, \qquad V_C \pm U_C = \begin{cases} (e^{y/p^2} \mp e^{-y/p^2})\sinh(\theta) & 2, 2' \\ (e^{y/p^2} \pm e^{-y/p^2})\cos(\theta) & II, III \end{cases}. \tag{B.2}$$

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Regions 1, 1', 2 and 2' are obtained from region II by[23]

$$
\begin{aligned}
(y,\theta)_{II} &\longrightarrow \begin{cases} (y \pm i\pi/2, \mp i\theta) & 1 \\ (y \pm i\pi/2, \pm i\theta) & 1' \end{cases} \\
(y/p^2, \pi/2 - \theta)_{II} &\longrightarrow \begin{cases} (y/p^2 \pm i\pi/2, \mp i\theta) & 2 \\ (y/p^2 \pm i\pi/2, \pm i\theta) & 2' \end{cases}
\end{aligned} \tag{B.3}
$$

Now one can start from any mode in region II and use relations (B.3) to define the modes in regions 1, 1', 2 and 2'. The global modes one obtains are analytic on the horizons.[24] If for example one defines a mode in region 1 by the transformation $(y,\theta)_{II} \to (y + i\pi/2, -i\theta)$ ($(y,\theta)_{II} \to (y - i\pi/2, i\theta)$) then this mode will be made from purely positive (negative) frequencies with respect to the affine parameter U_A.

If we properly use this modes to quantize the field separately in regions 1 and 1' or in regions 2 and 2' we will observe the HH vacuum of those regions. But, as proved in subsection (2.3), such a global Fock vacuum does not exist.

C. Hartle-Hawking quantization with $\epsilon = 0$

In this appendix we give some details regarding the quantization of a scalar field from the point of view of section 3.2 (quantization 2). A scalar field at hand is characterized by a representation of the parent $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ which is labelled by the Casimir eigenvalues j and ϵ. Here we consider only the continuous representations with $\epsilon = 0$. We construct the Hartle-Hawking (HH) vacuum in the regions of type A and show that it does not match with the analog of the HH vacuum in the regions of type C. We will conclude that for $\epsilon = 0$ there is no Fock vacuum in which there is a thermal distribution in regions of type A and C with their corresponding temperatures. The generalization for any ϵ is straightforward.

A basis of wave functions in region 1 which vanish in region 1' and which are eigenfunctions of the Killing vector ξ in (2.42) with eigenvalue ω is given by

$$
K_{-+}(\lambda,\mu;j;g)\,, \qquad K_{-+}(-\lambda - 2j, -\mu - 2j; j; g)^*\,, \tag{C.1}
$$

where

$$
\lambda - \mu = -i\frac{\omega}{\kappa_1}\,, \qquad \lambda + \mu = -2j = 1 - 2is\,. \tag{C.2}
$$

$K_{\pm\pm}$ are matrix elements of the representation characterized by $\epsilon = 0$ and j (the explicit form of these matrix elements can be found in [36]), κ_1 is the surface gravity of the outer horizon and we take $\omega, s > 0$. These modes are not orthogonal to each other in the Klein-Gordon norm (3.4). An orthogonal basis is given by the following linear combinations of

[23]This coordinate transformations descend from the parameterizations of the corresponding regions in $\mathrm{SL}(2,\mathbb{R})$.

[24]To be more precise, the modes have branch cuts in the complex planes so one has to specify exactly how to go from one point to the other. For example the modes in region 1 are obtained from a mode in region II by the analytic continuation $(\theta, y)(\gamma) = (\theta e^{-i\frac{\pi}{2}\gamma}, y + i\gamma\pi/2)$, where γ is a real parameter that runs from 0 to 1.

the modes (C.1):

$$F^+_{\omega,1} \propto K_{-+}(\lambda,\mu;j;g) + \frac{B(\lambda,1-\mu)}{B(-\lambda-2j,1+\mu+2j)^*}K_{-+}(-\lambda-2j,-\mu-2j;j;g)^*\,, \quad (C.3)$$
$$F^+_{\omega,2} \propto K_{-+}(\lambda,\mu;j;g) + \frac{B(\lambda,2j+1)+B(-\lambda-2j,2j+1)}{B(\lambda,2j+1)^*+B(-\lambda-2j,2j+1)^*}K_{-+}(-\lambda-2\tau,-\mu-2\tau;j;g)^*\,.$$

These modes are orthogonal since $F^+_{\omega,1}$ vanish at the past null infinity and $F^+_{\omega,2}$ vanish at the past horizon and the Klein-Gordon norm can be calculated at the past infinity of region 1. The corresponding images of these modes in region 1' ($F^-_{\omega,1}$ and $F^-_{\omega,2}$) are given by interchanging K_{-+} and K_{+-}. We define the following linear combinations

$$\begin{aligned} H^+_{\omega,i} &= \cosh(\phi_1)F^+_{\omega,i} + \sinh(\phi_1)F^-_{\omega,i}\,, \\ H^-_{\omega,i} &= \sinh(\phi_1)F^+_{\omega,i} + \cosh(\phi_1)F^-_{\omega,i}\,, \end{aligned} \quad (C.4)$$

where $\tanh(\phi) = e^{-\pi\frac{\omega}{\kappa_1}}$ and $i=1,2$. According to [34, 13] $H^+_{\omega,i}$ ($H^-_{\omega,i}$) have purely positive (negative) frequencies with respect to the affine parameters along the bifurcate Killing horizon of regions 1 and 1'. After the scalar field is quantized the HH vacuum of regions 1 and 1' is obtained if we associate an annihilation operator $a^{(+)}_{\omega,i}$ to $H^+_{\omega,i}$ and a creation operator $(a^{(-)}_{\omega,i})^\dagger$ to $H^-_{\omega,i}$ where $i=1,2$. In terms of the modes $H^\pm$ the quantum field ϕ is given by:[25]

$$\Phi_\omega(x) = \sum_{i=1}^{2}\left(H^+_{\omega,i}a^{(+)}_{\omega,i} + H^-_{\omega,i}\left(a^{(-)}_{\omega,i}\right)^\dagger + \text{h.c.}\right)\,. \quad (C.5)$$

To reproduce an analogous construction in regions 2 and 2' we start with the modes

$$K_{++}(\lambda,\mu;j;g)\,, \qquad K_{++}(-\lambda-2j,-\mu-2j;j;g)^*\,, \quad (C.6)$$

which are eigenfunctions of the Killing vector with eigenvalues ω. These modes form a basis of wave functions in region 2' and vanish in region 2. In parallel with (C.4) we obtain purely positive (negative) frequency modes $\tilde{H}^+_{\omega,j}$ ($\tilde{H}^-_{\omega,j}$) relative to the affine parameters along the inner horizon. These modes are given by linear combinations of (C.6) and their images

$$K_{--}(\lambda,\mu;j;g)\,, \qquad K_{--}(-\lambda-2j,-\mu-2j;j;g)^*, \quad (C.7)$$

with the thermal factor $\tanh(\phi_2) = e^{-\pi\frac{\omega}{\kappa_2}}$. Here κ_2 is the surface gravity of the inner horizon. For a given ϵ there is an (ϵ-dependent) non-trivial Bogolubov transformation between the modes of regions 1 and 1', and the modes of regions 2 and 2'. To show that this Bogolubov transformation is not trivial let us examine a mode

$$Y^+ = \cosh(\phi_1)K_{-+} + \sinh(\phi_1)K_{+-}\,, \quad (C.8)$$

which has purely positive frequencies with respect to the affine parameters along the outer horizon. Let $U>0$ be the affine parameter along the Killing horizon between regions 2'

[25]There is no sum over ω since different ω's correspond to different fields.

and III, and $U < 0$ the affine parameter along the Killing horizon between regions 2 and II. The restrictions of K_{-+} and K_{+-} to the horizon are

$$\begin{aligned} K_{-+}(\lambda,\mu;j;U)_{|U<0} &= \frac{(-1)^{2\epsilon}}{2\pi i} B(1+\mu+2j,-\lambda-\mu-2j)|U|^{-i\frac{\omega}{\kappa_2}} \\ K_{-+}(\lambda,\mu;j;U)_{|U>0} &= \frac{(-1)^{2\epsilon}}{2\pi i} B(\lambda,-\lambda-\mu-2j)U^{-i\frac{\omega}{\kappa_2}} \\ K_{+-}(\lambda,\mu;j;U)_{|U<0} &= \frac{1}{2\pi i} B(\lambda,-\lambda-\mu-2j)|U|^{-i\frac{\omega}{\kappa_2}} \\ K_{+-}(\lambda,\mu;j;U)_{|U>0} &= \frac{1}{2\pi i} B(1+\mu+2j,-\lambda-\mu-2j)U^{-i\frac{\omega}{\kappa_2}} \end{aligned} \tag{C.9}$$

So[26]

$$\begin{aligned} Y^{+}(\lambda,\mu;j;U) \propto\ & \theta(-U)|U|^{-i\frac{\omega}{\kappa_2}} + \\ & + \frac{B(\lambda,-\lambda-\mu-2j)+e^{-\pi\frac{\omega}{\kappa_1}}B(1+\mu+2j,-\lambda-\mu-2j)}{B(1+\mu+2j,-\lambda-\mu-2j)+e^{-\pi\frac{\omega}{\kappa_1}}B(\lambda,-\lambda-\mu-2j)}\theta(U)U^{-i\frac{\omega}{\kappa_2}} \end{aligned} \tag{C.10}$$

is not analytic on the lower half of the complex U plane and, therefore, it contains also negative frequencies with respect to U. We conclude that the Bogolubov transformation at hand is not trivial and the corresponding vacuum is not thermal in regions of type C.

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